Negative Interest Rates in Japan
An Equilibrium Analysis

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Abstract

We derive conditions under which a profit maximizing commercial bank may be willing to lend a foreign currency to another commercial bank at a negative equilibrium interest rate. The analysis is motivated by recent occurrences of negative interest rates in the yen call market. Also, profitability of banking activities plays a prominent role in the mechanics of negative interest rates.

1 Introduction

In March 2001, the Bank of Japan decided to follow a liquidity policy in which the commercial banks' current accounts were raised continuously over time.[1] Contrasting sharply with the traditional economic understanding this

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[1] At the outset of the so called *Quantitative Easing Policy*, the required reserve level for Japanese Banks at the Bank of Japan was four trillion yen. Since then, the Bank of Japan has increased the respective reservers up to 30-35 trillion yen. see Maeda [8]

has effected nominal interbank short-term interest rates to drop consistently below zero.[2]

According to standard economic reasoning, ex-post real interest rates could be negative as a consequence of the market's false estimation of the inflation rate. However, ex-ante real as well as nominal interest rates should always be non-negative because individuals prefer holding cash rather than lending at negative rates. This notion of money holding is premised on the assumption that, first, it is the individual himself who physically holds the money, and, second, that the individual has a storage technology at his disposal, i.e. he can transfer the money's value over periods. But, in modern financial markets it can hardly be claimed that institutions hold all their money in a vault. Instead, the actual legal titles change, more often than not, much faster than the money's physical location. Then, the divergence of ownership and factual property gives rise to various operational and legal risks that lie beyond inflation or depreciation. This risk is often described as settlement risk and it need not be due to default of the institution that holds the money physically. It can also include the risk of transactions not being effected and, as a consequence, one might not be able to meet claims that other parties have against oneself. As an example, consider timing differences in the payment as a result of the computer systems used in the clearing process. The Year-2000 problem was a main concern for many institutions such as banks and insurance companies. Hence, it seems sensible to concede that even cash holdings might contain a specific element or risk. As a consequence, without further assumptions, the above line of argument is less compelling and questionable.

In this paper we identify a set of conditions such that a commercial bank may indeed have an economic incentive to lend money at a negative (nominal) interest rate. The basic idea is in line with recent descriptions of the mechanics underlying negative interest rates as proposed by Maeda [8], Nishioka

[2]For a more detailed empirical description see Nishioka and Baba [9].

and Baba [9]. Maeda gives explicitly an empirically evidenced explanation for negative interest rates in the U.S. dollar-yen FX market which points on the possibility of riskless yen founding and differences in creditworthiness between Japanese and foreign banks.[3] Nishioka and Baba decompose the yen funding cost in the FX swap market into the yen riskfree interest rate, the credit risk premium for foreign banks, and the difference in the credit-risk premium for domestic banks between the yen and the U.S. dollar markets. Then, in combination with a low yen riskfree interest rate, negative yen funding costs for foreign banks may be due to the fact that the credit-risk premium for domestic banks is higher in the U.S. dollar market than in the yen market. This provides the backdrop for our model.

With this paper, we aim at contributing to the discussion by reformulating their case study into a general equilibrium analysis with profit maximizing banks. The story's baseline is the following: If a central bank starts to inject a large amount of liquidity into a banking sector which may encompass banks of lower than prime ratings, this is essentially equivalent to providing a free guarantee in the form of excessive currency collateral. This holds, since the currency has an economic value that a bank can rely on in all its transactions almost without limits and risks. Then, commercial banks with access to international currency markets may decide to swap the cheap collateral currency into another currency of international standing such as the U.S. dollar. Consequently, it could be reasonable for a bank to accept disadvantageous conditions if these swaps can be used to circumvent the bank's own lower than prime rating.[4] As a result, the cheap collateral currency accumulates on the current account balances of foreign banks. Given that these

[3]Foreign Exchange markets allow the market participants to exchange one currency for another.

[4]Note, that many swap type transactions are off-balance so that notorious institutions might have an incentive to shift away from their usual activities to more risky but highly profitable positions. Indeed, literature on the economics of swaps view them as a mechanism for maneuvering profitably around imperfections that might arise from different jurisdiction (e.g. regulation, accounting rules or taxation) or asymmetric information.

holdings are physically located at the central bank, involved foreign banks, however, attach a risk premium to their foreign currency holdings. This risk induces the banks to diversify by lending the collateral liquidity out in the interbank market, potentially even at negative rates. Alternatively, instead of risk-averse behavior, lending out foreign currency could be motivated by credit limits due to regulations or other risk concerns. Thus, one of the factors contributing to the economic feasibility of negative interest rates is the modern credit money system.

To put it in the Japanese framework, foreign banks were able to make an almost risk-free profit by investing any yen founds thus raised in the Bank of Japan's current account at negligible costs. The fact is consistent with Baba et al.'s [2] description that from March 2004 to March 2004 foreign bank's holdings at the Bank of Japan increased from 0.1 trillion yen to 5.4 trillion yen, which amounts to almost one quarter of the overall holdings of the Bank of Japan's current account balances by all financial institutions. However, in the presence of credit lines on the dealings with the Bank of Japan, foreign banks are not allowed to accumulate current account balances without limit. As a consequence, excess amounts are released in the call market at negative interest rates.

Negative interest rates are not a new phenomenon. E.g., according to Cecchetti [3], U.S. Treasury bonds appeared to have negative interest rates during the aftermath of the Great Depression. In this case, the puzzle was due to an option attached to Treasury securities to buy another security in the future. For another example, during the 1970s, the Swiss National Bank offered loans at negative interest rates as a means of steering exchange rates, but only to foreigners (see Kugler and Rich [7]). Negative rates occurred also on certain U.S. Treasury security repurchase agreements from early August to mid-November of 2003. Fleming and Garbade [4] explain it to be a consequence of the scarcity of a specific collateral used in repurchase agreements. The mechanics underlying the Japanese experience is different from all these earlier occurrences of negative interest rates, yet, for they were all more or less temporary in nature. In contrast, since November 1998, Japanese interest rates have dropped repeatedly below zero and have even stayed below zero over longer periods of time.

In the formal analysis we consider a world comprised of two currency areas, in which risk averse commercial banks manage their on and off balance sheet positions in order to maximize profits. The modelling of an individual bank follows the tradition of Pyle [10] and Hart and Jaffee [5]. Commercial banks may provide credits in both currencies. It is assumed that refinancing in the foreign currency is performed using currency swaps (e.g. because of a disadvantageous credit rating). Moreover, foreign currency may be deposited in the interbank market. We derive the individual banks' demand and supply curves in the involved financial markets, i.e., in the domestic credit market, in the foreign credit market, in the currency swap market, and in the call market for the collateral currency. From this, the equilibrium rate of return in the swap market is derived.

The remainder of this paper is organized as follows: In Section 2 we set up the model. The banks and their respective decision problems are analyzed. From this the equilibrium prices are computed and our main result is stated. Section 3 concludes.

2 The Model

There exist two different currency areas A and B. In both currency areas price-taking banks manage their portfolios over one period of time such as to maximize their profits under special considerations of the associated riskiness. For the sake of tractability and in order to be able to focus on the 'institutional riskiness' between different currency areas, we assume that the exchange rate e (units of area A per unit area B) is fixed and therefore does not add to the riskiness of international engagements. This could be justified by a sufficiently short period of time. All banks are faced with a demand for credits denominated in both currencies. They can refinance any liquidity that is denominated in their home currency through transactions with the central bank of their respective currency area. In contrast, for a bank to lend money in a foreign currency it must raise these funds by means of a currency swap. Basically, a currency swap is a financial agreement between two parties referring to the exchange of two specific amounts of two different currencies at the beginning and the repayment of the principal plus possible additional payments until the end of a specified period of time.[5] Since the performance failure by one party does not relieve the other party of its obligations, swaps are usually considered as limited in their credit risks. In our model, we assume that any transactions take place only at the beginning or at the end of the period. Furthermore, it is assumed that, since banks take explicitly into account the various risks that are associated with their contractual counterparts, banks require currency swaps to be paid on the spot. More precisely, assume that only currency B is risky. Therefore, in order to buy currency A by means of a currency swap, banks in currency area B must provide a sufficient amount of their home currency as collateral and, in addition, have to accept a discount which is due to be paid on the upfront.

More precisely, assume that there are three banks: Bank 1 and Bank 3 are located in currency area A, and Bank 2 is located in currency area B. Only Bank 1 and Bank 2 have access to the currency swap market.

Assumption 1 *In contrast to Bank 1 and Bank 2, Bank 3 has no access to the swap market.*

[5]The design of the repayment can vary considerably making the valuation of swaps somewhat challenging. As an example, the additional payments might include interest rates, or the repayment might either occur constantly over the period or directly at the maturity.

In the following super- or subscripts $i = 1, 2, 3$ or A, B identify the bank and currency area, respectively. Let $\tilde{\pi}_i$ denote Bank i's profit. Banks evaluate the mean-variance utility EU that can be associated with the random profit $\tilde{\pi}_i$ according to

$$E\mathrm{U}(\tilde{\pi}_i) \quad = \quad \mathbb{E}[\tilde{\pi}_i] - \frac{\beta_i}{2}\mathrm{Var}[\tilde{\pi}_i], \tag{1}$$

where $\mathbb{E}[\tilde{\pi}_i]$ is the expectation of profit $\tilde{\pi}_i$, $\mathrm{Var}[\tilde{\pi}_i]$ its variance and $\beta_i > 0$ is a parameter of risk aversion. For the ease of presentation, we assume that all random variables are uncorrelated and any rate of returns or costs are net.

Bank 1 Bank 1 operates in area A managing its credit portfolio which encompasses a supply X_1^A of credits in home currency A at rate $\tilde{r}_A$ and an amount of X_1^B of credits at rate $\tilde{r}_B$ that are denominated in the foreign currency B. While Bank 1 can always borrow liquidity from its central bank at rate $\rho_A \geq 0$, in order to provide a loan in currency B, it must swap an amount S of currency A: Bank 1 exchanges currency A for currency B with Bank 2, and, at the same time, agrees to reverse the exchange at the end of the period. The currency exchange is done at the prevailing market exchange rate $e > 1$. Although e is fixed, from Bank 1's point of view currency B is a risky asset. Its riskiness stems from the fact that bank 1's liquidity is not physically at its disposal. Instead, it is deposited at the bank's current account balance at the central bank of area B. As a consequence of an operational failure that is not further specified, assume that with a small non-zero probability q the central bank of currency area B may be unable to perform the transfers of funds as requested by its customers.[6] More precisely,

[6]Rather than a complete failure one could also think of a delay. However, if Bank 1 is obliged to deliver currency B such a risk of delay amounts to the risk of procuring currency B from another source under the potential of a generally distressed situation.

any currency B reserves yield an uncertain rate $\tilde{r}_L$ according to

$$\tilde{r}_L = \begin{cases} -1 & \text{with probability } q \\ 0 & \text{with probability } 1-q. \end{cases}$$

Now, the riskiness or r_L induces Bank 1, as a risk averse entity, to credit parts of this liquidity to its costumers or to sell it to Bank 3 at rate r_Z. For the swap to be effected its price must include this risk. In particular, let $i_S S$ be the price for an amount of swap S, which we assume that Bank 2 pays it upfront. With this, for Bank 1 the only relevant risk of the swap is due to its liquidity holdings in currency B. So far, we assume that all this liquidity is deposited at the currency B central bank and, under no circumstances, will it be withdrawn.

Finally, taking into account that the amount swapped, S, equals the amount credited in currency B, i.e. eX_1^B, plus the liquidity hold in currency B, i.e. eL_1^B, and the amount eZ_1^B of currency B that is sold to Bank 3 on the call market, Bank 1's profit can be written as:

$$\begin{aligned} \tilde{\pi}_1 &= \underbrace{\tilde{r}_A X_1^A + i_S S}_{\substack{\text{returns on investments} \\ \text{denominated in currency } A}} + \underbrace{e\left(\tilde{r}_B X_1^B + \tilde{r}_L L_1^B\right)}_{\substack{\text{returns on investments} \\ \text{denominated in currency } B}} \\ &- \underbrace{\rho_A\left(X_1^A + (1-i_S)S - C_1\right)}_{\text{costs of funding in currency } A} + \underbrace{r_Z e\left(\tfrac{S}{e} - X_1^B - L_1^B\right)}_{\text{return from deposits in currency } B}, \quad (2) \end{aligned}$$

with decision variables X_1^A, X_1^B, S, L_1^B. The resulting first order conditions of the optimization problem as given by the utility function (1) and profit equation (2) allow us to state the following No-Arbitrage-Condition.

Theorem 1 *A necessary condition for the call market and the swap market* *to be free of arbitrage is*

$$r_Z + i_S \quad = \quad \rho_A(1 - i_S), \tag{3}$$

where r_Z is the expected rate of return of the call market.

<u>PROOF</u> The equation is directly implied by Bank 1's first order condition concerning the amount swapped S.

□

From Bank 1's perspective equation (3) compares interest income and interest payment resulting from a swap transaction. The term on the right-hand side is what Bank 1 has to pay in order to borrow currency A from its central bank to finance one unit of swap. This unit of swap yields directly the upfront rate i_S and, in addition, r_Z when lent out in the call market. To see the reason for why equation (3) must hold, assume $r_Z > \rho_A(1 - i_S) - i_S$. Then, Bank 1 would increase its activities in the swap market and deposit the additional amount of currency B in the call market, earning a positive margin. On the other hand, if r_Z is smaller than $\rho_A(1 - i_S) - i_S$, then Bank 1 would reduce its activity in the swap market. In an extreme scenario Bank 1 would even want to revert the swap flow, thereby receiving currency A against currency B and use the additional currency A in order to reduce funding from central bank in currency area A. Rewriting equation (3) yields

$$r_Z \quad = \quad \rho_A(1 - i_S) - i_S.$$

This relation clarifies the role of the swap rate in the mechanics of negative interest rates in the call market. If i_S is high enough, rates on the call market are forced below zero. Indeed, if i_S increases above $\rho_A/(1 + \rho_A)$, then no-arbitrage would imply negative interest rates. Indeed, Theorem 1 is in line with the the empirical observation that, in Japan, negative interest rates were accompanied by extreme swap rates that Japanese banks had to pay on

Dollar swaps.

Computing the optimal values one obtains

$$\overset{*}{X}{}_1^A = \frac{r_A - \rho_A}{\beta_1 \sigma_A^2} \tag{4}$$

$$\overset{*}{X}{}_1^B = \frac{r_B - r_Z}{e\beta_1 \sigma_B^2} \tag{5}$$

$$\overset{*}{L}{}_1^B = \frac{r_L - r_Z}{e\beta_1 \sigma_L^2}, \tag{6}$$

where r_B, r_L, r_Z denote expected values of $\tilde{r}_B, \tilde{r}_L, \tilde{r}_Z$, respectively. Thus, due to the assumed independence of asset returns, Bank 1 determines the optimal positions independently from each other. Note that, in equation (5) and (6) the funding rate is linked to the interest rate ρ_A and to the swap rate i_S via Theorem 1. Moreover, since $r_L = -q$ Bank 1 holds a positive amount of currency B liquidity only if $r_Z < r_L$, that is, lending in the call market is less profitable than holding cash.

Bank 2's accounting unit is currency B. At a given rate $\tilde{r}_B$ it supplies credits X_2^B which are denominated in currency B. Further, by offering an amount X_2^A, Bank 2 tries to meet the demand for credits that are denominated in units of currency A at an uncertain rate $\tilde{r}_A$. For doing so, it has to swap with Bank 1 for the respective currency. However, Bank 2 cannot simply buy at the prevailing exchange rate e an amount of the respective currency that equals to what it intends to credit. Instead it must accept a certain discount rate i_S which can be considered as a price for the swap and that must be paid upfront. Thus,

$$X_2^A = (1 - i_S)S. \tag{7}$$

In order to clear all its transactions, when the overall amount credited exceeds the bank's endowment C_2 in currency B, Bank 2 has to to rely on additional cash which it borrows from the central bank at rate ρ_B. For this, we make the additional assumption which we shall drop in the extension:

Assumption 2 *Bank 1 and Bank 3 do not offer any uncollateralized credit to Bank 2.*

Similarly to Bank 1, Bank 2's profit $\tilde{\pi}_2$ at the end of the period can be formalized as

$$\begin{aligned}\tilde{\pi}_2 \quad = \quad & \underbrace{\frac{\tilde{r}_A X_2^A}{e}}_{\substack{\text{returns on investments}\\ \text{denominated in currency } A}} \quad + \quad \underbrace{\tilde{r}_B X_2^B}_{\substack{\text{returns on investments}\\ \text{denominated in currency } A}} \\ & - \underbrace{\frac{i_S S}{e}}_{\text{costs of funding in currency } A} - \underbrace{\rho_B \left(X_2^B + \frac{S}{e} - C_2 \right)}_{\text{return from deposits in currency } B} \quad . \qquad (8)\end{aligned}$$

The first term expresses the gross benefit from lending an amount of X_2^A at rate $\tilde{r}_A$ to some customers. Since the yields are denominated in currency A they have to be translated in currency B by multiplying with $1/e$. The second term is the benefit that is gained by crediting an amount of X_2^B at rate $\tilde{r}_B$. The costs for the swap are given by the third term, and, finally, the term in brackets indicates the costs that occur if the bank borrows money from the central bank to refinance its activities.

Then, from the resulting first order conditions of the optimization problem given by the utility function (1) an optimal supply of credits $\overset{*}{X}^A, \overset{*}{X}^B$

can be derived

$$\overset{*}{X}{}_2^A = \frac{e}{\beta_2 \sigma_A^2} \left(r_A - \frac{i_S + \rho_B}{1 - i_S} \right) \tag{9}$$

$$\overset{*}{X}{}_2^B = \frac{r_B - \rho_B}{\beta_2 \sigma_B^2}. \tag{10}$$

Note that, $\overset{*}{X}{}_2^A$ is Bank 2's demand for currency A and that Bank 2 does not hold any excess liquidity in currency A. This holds because of our assumption that currency A is not risky and that no additional investment opportunities exist. So, holding cash in currency A would only induces indirect costs in form of the costs for rising the equivalent currency B amount from the central bank, and direct costs from paying the swap price i_S. So, left with no profitable investment opportunity, Bank 2 only would have to bear costs, thereby reducing its expected profit.

Bank 3 supplies credits X_3^A in its home currency at rate $\tilde{r}_A$ and credits X_3^B at rate $\tilde{r}_B$ which are cleared in the foreign currency B. It finances its home currency A transactions through own funds C_3 or through borrowing money from the central bank at cost ρ_A. Thus, it borrows currency B from Bank 1 at rate r_Z. Analogously to Bank 1 and 3, Bank 2's profit is:

$$\begin{aligned} \tilde{\pi}_3 = {} & \underbrace{\tilde{r}_A X_3^A}_{\substack{\text{returns on investments} \\ \text{denominated in currency } A}} + \underbrace{e\left(\tilde{r}_B X_3^B + \tilde{r}_L L_3^B\right)}_{\substack{\text{returns on investments} \\ \text{denominated in currency } B}} \\ & - \underbrace{\rho_A \left(X_3^A - C_3\right)}_{\text{costs of funding in currency } A} - \underbrace{r_Z e \left(L_3^B + X_3^B\right)}_{\text{return from deposits in currency } B} \end{aligned} \tag{11}$$

From the resulting first order conditions one can derive the supply of credits and the demand for currency B, defined as $Z_3^B = X_3^B + L_3^B$. So,

$$\overset{*}{X}{}_3^A = \frac{r_A - \rho_A}{\beta_3 \sigma_A^2} \tag{12}$$

$$\overset{*}{X}{}_3^B = \frac{r_B - r_Z}{e \beta_3 \sigma_B^2} \tag{13}$$

$$\overset{*}{L}{}_3^B = \frac{r_L - r_Z}{e \beta_3 \sigma_L^2}. \tag{14}$$

This allows to calculate Z_3^B, since $Z_3^B = X_3^B + L_3^B$.

Equilibrium The demand and supply functions and the condition in Theorem 1 that we have obtained so far can be used to calculate expressions for the expected equilibrium price r_Z. Supply must equal demand both on the swap and on the call market. As stated above, Bank 1's supply of currency A in the swap market, created by a demand in currency B, is

$$\frac{\overset{*}{S}}{e} = \overset{*}{X}{}_1^B + \overset{*}{L}{}_1^B + \overset{*}{Z}{}_1^B,$$

with $\overset{*}{Z}{}_1^B$ indicating the utility maximizing amount of currency B that is lend on the call market to Bank 3. Bank 2's demand for currency A is given by $\overset{*}{X}{}_2^B$ and Bank 3's demand for currency B, i.e. $\overset{*}{Z}{}_3^B$, equals its supply of credits in currency B, that is $\overset{*}{X}{}_3^B$, and its liquidity reserve in currency B, $\overset{*}{L}{}_3^B$. Putting these equations together results in

$$\frac{\overset{*}{X}{}_2^A}{(1 - i_S)e} = \overset{*}{X}{}_1^B + \overset{*}{L}{}_1^B + \overset{*}{X}{}_3^B + \overset{*}{L}{}_3^B. \tag{15}$$

Now, by plugging in all optimal demand and supply decisions we obtain our main result.

Note that the currency swap, by construction, is a riskfree transaction from the perspective of Bank 1. Indeed, the swap rate is paid upfront in the form of a discount on the outstanding loan in currency A. Thus, if Bank 2 should default within the term of the currency swap then Bank 1 would end up with the collateral S/e which can be exchanged for an equivalent amount of currency A. The risk resulting from the swap transaction is therefore determined exclusively by the investments undertaken with the help of the resulting currency holdings, i.e. the risks resulting from either keeping currency B on the reserve account or the risk from credit investments in currency B.

Theorem 2 *Assume that ρ_B is closed to zero. For given expected rates r_A, r_B, r_L and given costs ρ_A, ρ_B in equilibrium it holds that:*

$$r_Z \quad \approx \quad -q + \frac{qr_B}{\sigma_B^2} - \frac{q\hat{\beta}}{\sigma_A^2}\left(r_A - \rho_A - \rho_B\right),$$

where

$$\hat{\beta} := \frac{\frac{e}{\beta_2}}{\frac{1}{\beta_1} + \frac{1}{\beta_3}}.$$

can be seen as the aggregate risk-aversion.

<u>Proof</u> see Appendix □

It may not be obvious to the reader why Bank 1 should have an interest in holding liquidity in the foreign currency. After all, given that this liquidity is effectively a deposit with a foreign central bank, any holdings in the foreign currency are a risky asset with a negative expected return from the perspective of the commercial bank. However, the economic reason for Bank 1 to hold cash in the foreign currency is the absence of a profitable alternative. One alternative is to lend the cash holdings out in the interbank market but if are that is negative this may not be a profitable alternative. Another alternative for reducing cash holdings in the foreign currency could be to reduce currency swap trading vis-a-vis foreign banks. This, however, would cost Bank 1 an opportunity cost of i_S. As a consequence, the best of all these alternatives can be to hold liquid means in the foreign currency.

Note that, if the probability of settlement failure of the area B central bank q tends to zero, in Theorem 2 all terms disappear. Hence, in the case of $q = 0$ the call market rate is also zero, i.e. $r_Z = 0$. If we assume that the ratio between q and σ_A^2 and σ_B^2 is very small, the main influence on the interest rate r_Z is due to the probability q. Moreover, as one would expect, a higher rate r_B on credits in currency B has a positive effect on r_Z. The underlying reason is quite intuitive. A more profitable rate of currency B credits does not affect Bank 2's swap demand while, at the same time, it leads to a higher demand for currency B on the part of Bank 1 and Bank 3. As a consequence, to rise more currency B through the swap market, its price i_S must decrease. In line with Theorem 1, this amounts to an increase in r_Z. A similar argument holds in the case of a higher r_A. Credits in currency A get more profitable, with the consequence that, on one hand, Bank 1 and Bank 3 are relatively less interested in currency B credits thus reducing the demand for currency B. On the other hand, Bank 2 would like to expand its activities in currency A credits which drives up its swap demand. The net effect is a higher i_S and a lower r_Z.

ρ_A and ρ_B have a positive influence on the interest rate r_Z since they reflect the costs of refinancing currency A and B. If any refinancing becomes more expansive, this effects a lower supply or demand on the swap markets, which, in turn, leads to a reduced supply of currency B on the swap market.

In Theorem 2 it was assumed that the refinancing rate offered by the central bank of currency area B should be sufficiently low. To see the rational for the assumption consider an extension of the model in which we an additional bank in currency area B. These banks will seek appropriate funding for their investments in their domestic currency B. The straightforward alternative is the funding at rate ρ_B from the central bank. Another alternative, however, is to exploit the potentially more attractive conditions in the currency B call market. However, in order to convince in currency area A to offer an uncollateralized loan to banks operating in currency B a credit spread must be paid. refinancing in the call market will therefore only be attractive for a bank operating in currency area B if the difference between ρ_B and r_Z exceeds the credit spread.

Thus, if ρ_B is high then currency B becomes to expensive to serve as a collateral currency such that the difference between ρ_B and r_Z should decline. If this is the case then a change in the refinancing behavior of banks operating in currency area B should occur and the conditions in the call market should coincide with the central banks funding rate ρ_B.

3 Conclusion

We developed a model in which risk averse banks manage portfolios consisting of assets denominated in different currencies. For refinancing the investments banks acquire foreign currency on an international swap market. Supply and demand functions were derived for each bank's decision problem. This allowed us to state explicitly the equilibrium price for the call market as a function of central banks' funding conditions and the risk return characteristics of credit portfolios in the two currency areas. In particular, we derived a set of economic conditions that imply negative interbank rates for foreign central bank reserves.

Our conclusions differ from those obtained by Nishioka and Baba 2004 [9] (henceforth NB), who impose a no-arbitrage condition between alternative funding sources for US-Dollars. Specifically, NB assume that an individual commercial bank is indifferent between raising US-Dollars directly from the interbank markets and exchanging yen for Dollar in the currency swap market. Our analysis suggests that this no-arbitrage condition may not be binding. Indeed, as a loan denominated in US-Dollars implies costs that depend on the rating of the bank while a currency swap does not depend on this rating, it is not necessarily the case that this no-arbitrage condition is satisfied. In fact, we would deem it more plausible to assume that any commercial bank has recourse to the channel of refinancing that implies the lowest funding costs.

Appendix

Proof of Theorem 2: The equilibrium in the swap market is characterized by the condition that supply equals demand, that is

$$\begin{aligned} \frac{S_1}{e} &= S_2 \\ e(L_1^B + X_1^B + Z_1^B) &= \frac{X_2^A}{1 - i_S}. \end{aligned}$$

Hence, with $Z_1^B = Z_3^B = X_3^B + L_3^B$ (equilibrium condition in the call market) and Theorem 1, one obtains

$$\begin{aligned} r_L \frac{1}{\sigma_L^2} + r_B \frac{1}{\sigma_B^2} - (1 + r_Z) \left(\frac{1}{\sigma_B^2} + \frac{1}{\sigma_L^2} \right) &= \\ \hat{\beta} \frac{1}{\sigma_A^2} \frac{1 + \rho_A}{1 + r_Z} \left(1 + r_A - \left(\frac{1 + \rho_A}{1 + r_Z} \right) (1 + \rho_B) \right) - \left(\frac{1}{\sigma_B^2} + \frac{1}{\sigma_L^2} \right). \end{aligned}$$

This can be rewritten as a polynomial in $(1+r_Z)$ of degree 3. So, rearranging terms and substituting r_L by $-q$ and $\sigma_L^2 = q(1-q)$, one obtains

$$\begin{aligned}\left(1+\frac{q(1-q)}{\sigma_B^2}\right)(1+r_Z)^3 - \left((1-q)+\frac{q(1-q)(1+r_B)}{\sigma_B^2}\right)(1+r_Z)^2 \\ + \quad \frac{\hat{\beta}q(1-q)(1+\rho_A)(1+r_A)}{\sigma_A^2}(1+r_Z) \qquad (*) \\ - \quad \frac{\hat{\beta}q(1-q)(1+\rho_A)^2(1+\rho_B)}{\sigma_A^2} \quad = \quad 0.\end{aligned}$$

The resulting equilibrium rate $\overset{*}{r}_Z$ must be solution of equation (*). Think of equation (*) as an implicit function $F(r_Z, q) = 0$ with. Note that (*) implies for $q = 0$ that $r_Z = 0$. Now, develop equation (*) in a Taylor's series around the point $(r_Z = 0, q = 0)$ which is possible since (*) is well-defined in any ε-neighborhood of this point. Besides, note that the resulting Taylor's series is finite. Nevertheless, we neglect terms of higher order. Hence

$$0 \quad \approx \quad \left.\frac{\mathrm{d}F}{\mathrm{d}r_Z}\right|_{\substack{r_Z = 0 \\ q = 0}} \quad (r_Z - r_L) + \left.\frac{\mathrm{d}F}{\mathrm{d}\sigma_L^2}\right|_{\substack{r_Z = 0 \\ q = 0}} \quad \sigma_L^2.$$

The corresponding expressions can be calculated from (*) leading to the equation

$$\begin{aligned} r_Z \quad \approx \quad -q \quad + \quad & q\frac{r_B}{\sigma_B^2} \\ - \quad & q\frac{\hat{\beta}}{\sigma_A^2}\left((1+r_A)(1+\rho_A) - (1+\rho_A)^2(1+\rho_B)\right) - R,\end{aligned}$$

where R denotes the higher terms. Now, by substituting $(1+r_A)(1+\rho_A)$ through the approximation $1+r_A+\rho_A$ and $(1+\rho_A)^2(1+\rho_B)$ through $1+2\rho_A+\rho_B$, and approximating the fractions $1/(1+r_L) \approx r_L$ and $1/(1+r_L)^2 \approx 2r_L$ the last equation can be approximated as

$$r_Z \quad \approx \quad -q \quad + \quad q\frac{r_B}{\sigma_B^2} \quad - \quad q\frac{\hat{\beta}}{\sigma_A^2}\left(r_A - \rho_A - \rho_B\right),$$

so that the statement follows. □

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